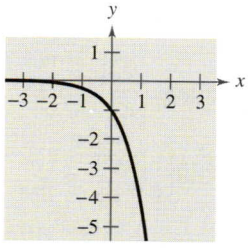
Review Exercises

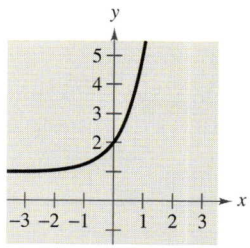
In Exercises 1–6, match the function with its graph. [The graphs are labeled (a) through (f).]

1. $f(x) = 4^x$
2. $f(x) = 4^{-x}$
3. $f(x) = -4^x$
4. $f(x) = 4^x + 1$
5. $f(x) = \log_4 x$
6. $f(x) = \log_4(x - 1)$

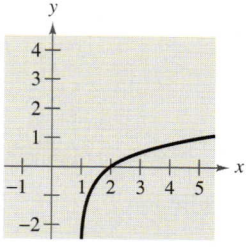
(a)



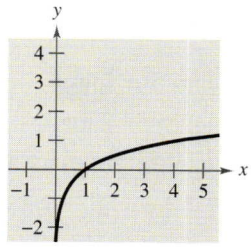
(b)



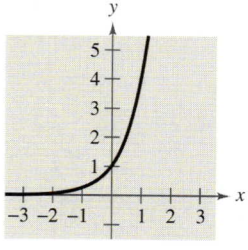
(c)



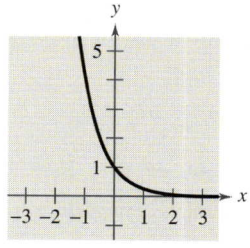
(d)



(e)



(f)



In Exercises 7–12, sketch the graph of the function.

7. $f(x) = 0.3^x$
8. $g(x) = 0.3^{-x}$
9. $h(x) = e^{-x/2}$
10. $h(x) = 2 - e^{-x/2}$
11. $f(x) = e^{x+2}$
12. $s(t) = 4e^{-2/t}, \quad t > 0$

In Exercises 13 and 14, use a graphing utility to graph the function. Identify any asymptotes.

13. $g(x) = 200e^{4/x}$
14. $f(x) = \dfrac{10}{1 + 2^{-0.05x}}$

In Exercises 15 and 16, complete the table to determine the balance A for P dollars invested at rate r for t years and compounded n times per year.

n	1	2	4	12	365	Continuous
A						

15. $P = \$3500$, $r = 10.5\%$, $t = 10$ years
16. $P = \$2000$, $r = 12\%$, $t = 30$ years

In Exercises 17 and 18, complete the table to determine the amount P that should be invested at rate r to produce a balance of \$200,000 in t years.

t	1	10	20	30	40	50
P						

17. $r = 8\%$, compounded continuously
18. $r = 10\%$, compounded monthly

19. ***Waiting Times*** The average time between incoming calls at a switchboard is 3 minutes. The probability of waiting less than t minutes until the next incoming call is approximated by the model

$$F(t) = 1 - e^{-t/3}.$$

If a call has just come in, find the probability that the next call will be within

(a) $\frac{1}{2}$ minute. (b) 2 minutes. (c) 5 minutes.

20. ***Depreciation*** After t years, the value of a car that cost \$14,000 is given by

$$V(t) = 14{,}000\left(\tfrac{3}{4}\right)^t.$$

(a) Use a graphing utility to graph the function.

(b) Find the value of the car 2 years after it was purchased.

(c) According to the model, when does the car depreciate most rapidly? Is this realistic? Explain.

21. ***Trust Fund*** On the day a person was born, a deposit of $50,000 was made in a trust fund that pays 8.75% interest, compounded continuously.

(a) Find the balance on the person's 35th birthday.

(b) How much longer would the person have to wait to get twice as much?

22. ***Fuel Efficiency*** A certain automobile gets 28 miles per gallon of gasoline for speeds up to 50 miles per hour. Over 50 miles per hour, the number of miles per gallon drops at a rate of 12% for each additional 10 miles per hour. If s is the speed and y is the number of miles per gallon, then

$$y = 28e^{0.6-0.012s}, \qquad s \geq 50.$$

Use this model to complete the table.

s	50	55	60	65	70
y					

In Exercises 23–28, sketch the graph of the function. Identify any asymptotes.

23. $g(x) = \log_2 x$

24. $g(x) = \log_5 x$

25. $f(x) = \ln x + 3$

26. $f(x) = \ln(x - 3)$

27. $h(x) = \ln(e^{x-1})$

28. $f(x) = \frac{1}{4} \ln x$

In Exercises 29 and 30, use a graphing utility to graph the function.

29. $y = \log_{10}(x^2 + 1)$

30. $y = \sqrt{x} \ln(x + 1)$

In Exercises 31 and 32, write the exponential equation in logarithmic form.

31. $4^3 = 64$

32. $25^{3/2} = 125$

In Exercises 33–36, evaluate the expression by hand.

33. $\log_{10} 1000$

34. $\log_9 3$

35. $\ln e^7$

36. $\log_a \frac{1}{a}$

In Exercises 37–40, evaluate the logarithm using the change-of-base formula. Do each problem twice, once with common logarithms and once with natural logarithms. Round the result to three decimal places.

37. $\log_4 9$

38. $\log_{1/2} 5$

39. $\log_{12} 200$

40. $\log_3 0.28$

In Exercises 41–44, use the properties of logarithms to write the expression as a sum, difference, and/or multiple of logarithms.

41. $\log_5 5x^2$

42. $\log_7 \dfrac{\sqrt{x}}{4}$

43. $\log_{10} \dfrac{5\sqrt{y}}{x^2}$

44. $\ln \left| \dfrac{x-1}{x+1} \right|$

In Exercises 45–48, write the expression as the logarithm of a single quantity.

45. $\log_2 5 + \log_2 x$

46. $\log_6 y - 2 \log_6 z$

47. $\frac{1}{2} \ln|2x - 1| - 2 \ln|x + 1|$

48. $5 \ln|x - 2| - \ln|x + 2| - 3 \ln|x|$

True or False? **In Exercises 49–54, determine whether the equation or statement is true or false.**

49. $\log_b b^{2x} = 2x$

50. $e^{x-1} = \dfrac{e^x}{e}$

51. $\ln(x + y) = \ln x + \ln y$

52. $\ln(x + y) = \ln(x \cdot y)$

53. $\log\left(\dfrac{10}{x}\right) = 1 - \log x$

54. The domain of the function $f(x) = \ln x$ is the set of all real numbers.

55. ***Snow Removal*** The number of miles s of roads cleared of snow is approximated by the model

$$s = 25 - \frac{13 \ln(h/12)}{\ln 3}, \qquad 2 \leq h \leq 15$$

where h is the depth of the snow in inches. Use this model to find s when $h = 10$ inches.

56. ***Climb Rate*** The time t, in minutes, for a small plane to climb to an altitude of h feet is given by

$$t = 50 \log_{10} \frac{18{,}000}{18{,}000 - h}$$

where 18,000 feet is the plane's absolute ceiling.

(a) Determine the domain of the function appropriate for the context of the problem.

(b) Use a graphing utility to graph the time function and identify any asymptotes.

(c) As the plane approaches its absolute ceiling, what can be said about the time required to further increase its altitude?

(d) Find the time for the plane to climb to an altitude of 4000 feet.

In Exercises 57–62, solve the exponential equation. Round your result to three decimal places.

57. $e^x = 12$

58. $e^{3x} = 25$

59. $3e^{-5x} = 132$

60. $14e^{3x+2} = 560$

61. $e^{2x} - 7e^x + 10 = 0$

62. $e^{2x} - 6e^x + 8 = 0$

In Exercises 63–68, solve the logarithmic equation. Round the result to three decimal places.

63. $\ln 3x = 8.2$

64. $2 \ln 4x = 15$

65. $\ln x - \ln 3 = 2$

66. $\ln\sqrt{x + 1} = 2$

67. $\log(x - 1) = \log(x - 2) - \log(x + 2)$

68. $\log(1 - x) = -1$

In Exercises 69–72, use a graphing utility to solve the equation. Round the result to two decimal places.

69. $2^{0.6x} - 3x = 0$

70. $25e^{-0.3x} = 12$

71. $2 \ln(x + 3) + 3x = 8$

72. $6 \log_{10}(x^2 + 1) - x = 0$

In Exercises 73 and 74, find the exponential function $y = ae^{bx}$ that passes through the points.

73. $(0, 2), (4, 3)$

74. $\left(0, \frac{1}{2}\right), (5, 5)$

75. ***Demand Function*** The demand equation for a certain product is given by

$$p = 500 - 0.5e^{0.004x}.$$

Find the demand x for a price of (a) $p = \$450$ and (b) $p = \$400$.

76. ***Typing Speed*** In a typing class, the average number of words per minute typed after t weeks of lessons was found to be

$$N = \frac{157}{1 + 5.4e^{-0.12t}}.$$

Find the time necessary to type (a) 50 words per minute and (b) 75 words per minute.

77. ***Compound Interest*** A deposit of $10,000 is made in a savings account for which the interest is compounded continuously. The balance will double in 5 years.

(a) What is the annual interest rate for this account?

(b) Find the balance after 1 year.

78. ***Sound Intensity*** The relationship between the number of decibels β and the intensity of a sound I in watts per square centimeter is given by

$$\beta = 10 \log_{10}\left(\frac{I}{10^{-16}}\right).$$

Determine the intensity of a sound in watts per square centimeter if the decibel level is 125.

79. ***Earthquake Magnitudes*** On the Richter scale, the magnitude R of an earthquake of intensity I is given by

$$R = \log_{10} \frac{I}{I_0}$$

where $I_0 = 1$ is the minimum intensity used for comparison. Find the intensity per unit of area for the following values of R.

(a) $R = 8.4$

(b) $R = 6.85$

(c) $R = 9.1$